Theory of Distributed Systems

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Exercise 4

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Due: 23.11.2021, **8:15h**

Please submit your solution in PDF format by sending an email to {schmalhofer,varricchio}@em.uni-frankfurt.de. Make sure that your solution reaches us before 8:15 am! Solutions are discussed on Nov 26th, 10:00h - 12:00h (Zoom Meeting-ID: 963 6309 6725, same password as lecture material).
Exercises with * are bonus; they count for your score but not for the sum of points.

Exercise 4.1. *ThreeColor2Matching on Trees* (4 Points)

Given a legal 3-coloring on a rooted tree, provide a deterministic algorithm that computes a maximal matching in time $O(1)$.

Exercise 4.2. *With High Probability* (3 Points)

Consider the following (more general) definition: an event X occurs with high probability (whp) if there exist fixed constants $n_0 \in \mathbb{N}$ and $c > 0$ such that $\Pr(X) \geq 1 - \frac{1}{n^c}$ holds for all $n \geq n_0$.
Show that if X and Y are two (not necessarily independent) events, each occurring with high probability, then $X \cap Y$ also occurs with high probability.

Exercise 4.3. *Number of Rounds: whp vs. expectation* (3 Points)

Consider a randomized distributed algorithm ALG running on an n-node network, and denote by T the (random) number of time steps used by ALG. Suppose ALG always terminates in at most n time steps, i.e., $\Pr(T \leq n) = 1$. Show that if $\Pr(T \leq \log_2 n) \geq 1 - 1/n$, then $\mathbb{E}[T] = \mathcal{O}(\log n)$.

Exercise 4.4. *Randomized Matching* (11 = 2 + 3 + 3 + 3 and 4* Points)

Given a bounded-degree graph $G = (V, E)$, i.e., the maximum degree Δ is upper bounded by a constant, consider the following randomized matching algorithm in the synchronous CONGEST model: Initially all nodes are active. In every phase, every active node selects uniformly at random one active neighbor and sends a match request to the latter. We say two nodes get matched when they mutually send each other match requests in a phase; once they get matched, they become inactive and inform all neighbors. If an active node learns all neighbors are inactive before it got matched, it becomes inactive and remains isolated.

a) Show that the algorithm outputs a maximal matching even if G is not a bounded-degree graph.

b) Prove that in every phase at least a constant fraction of edges is removed in expectation.

c) Prove that in every phase at least a constant fraction of nodes is removed in expectation.

d) Show that the algorithm terminates in $O(\log n)$ phases with high probability.

e*) Consider a run of the algorithm on an n-clique, which is no longer a bounded-degree graph. Show that the expected number of rounds is at least $c \cdot n$, for a constant $c > 0$.

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