Theory of Distributed Systems

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Prof. Dr. Martin Hoefer
Marco Schmalhofer, Giovanna Varricchio

JOHANN WOLFGANG GOETHE
UNIVERSITÄT
FRANKFURT AM MAIN

Institute of Computer Science
Algorithms und Complexity

Exercise 1

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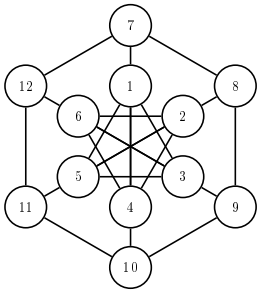
Due: 02.11.2021, **8:15h**

Please submit your solution in PDF format by sending an email to {schmalhofer,varricchio}@em.uni-frankfurt.de. Make sure that your solution reaches us before 8:15 am! Solutions are discussed on Nov 5th, 10:00h - 12:00h (Zoom Meeting-ID: 963 6309 6725, same password as lecture material).

Exercise 1.1. *Graph Terminology* (6 = 3 + 3 Points)

a) Consider the following graph $G = (V, E)$:



Determine $Diam(G)$, $Rad(G)$ and *all* centers of G.

b) Prove or disprove: There exists a constant $k \in \mathbb{N}$ such that for every undirected connected graph $G = (V, E)$ and every pair u, v of its centers, $dist_G(u, v) \leq k$.

Exercise 1.2. *Centers in Trees* (7 = 2 + 2 + 3 Points)

Let $T = (V, E)$ be any tree and C be the set of its centers.

a) Let $T' = (V', E')$ be the sub-tree obtained from T by removing all its leaves. Denote by C' the set of center nodes in T'. Show the following: If T' is non-empty, then $C = C'$.

b) Show that $|C| \leq 2$. You can use the previous statement.

c) Design a distributed algorithm in the synchronous CONGEST-model, which makes the center(s) aware to be center(s). Analyze time and message complexity of your algorithm.

Exercise 1.3. *Diameters of Trees* (6 Points)

Consider the synchronous CONGEST-model. We are given a rooted tree $T = (V, E)$, where node r_0 is root. Nodes are aware of being part of a tree and also the root node knows that it is the root.

Describe a distributed algorithm that calculates the diameter $Diam(T)$ of T. In particular, after termination of the algorithm, the root node r_0 should store the diameter $Diam(T)$.

Your algorithm should have time complexity $\mathcal{O}(Depth(T))$ and message complexity $\mathcal{O}(|V|)$.

The assignments and further information concerning the lecture can be found at
`http://algo.cs.uni-frankfurt.de/lehre/tds/winter2122/tds2122.shtml`

E-mail: `schmalhofer@em.uni-frankfurt.de, varricchio@em.uni-frankfurt.de`