Theory of Distributed Systems

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Prof. Dr. Martin Hoefer,
Niklas Hahn

JOHANN WOLFGANG GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Institute of Computer Science
Algorithms und Complexity

Exercise 10

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Exercise 10.1. *c-independence* (3 + 3 = 6 Points)

Show the following for an arbitrary unweighted graph G:

a) If G is c-independent, then G has independence number $\alpha(G) = \Theta(c)$.

b) If $\alpha(G) \leq c$, then G is c-independent.

Exercise 10.2. *Green IT* (2 + 2 + 4 = 8 Points)

Due to the continuous increase in energy costs, we want to penalize unsuccessful transmissions. To do so, we change the utility $u_i(x^t)$ gained for the chosen action. Now $\beta > 1$ is the new cost for an unsuccessful transmission:

$$u_i(x^t) = \begin{cases} 1 & x_i^t = 1 \text{ and } v_i \text{ successful} \\ -\beta & x_i^t = 1 \text{ and } v_i \text{ not successful} \\ 0 & x_i^t = 0 \end{cases}$$

a) Adapt Lemma 68 to the new scenario, i.e. prove the following lemma:

Lemma:
Suppose a history x is such that node v_i has regret $R_i(x) \leq 0$. Then at least a $\frac{\beta}{1+\beta}$-fraction of v_i's transmission attempts have been successful.

Use the lemma from a) to show a result similar to Theorem 38 for the new scenario:

Theorem:
Consider a c-independent conflict graph. Suppose there is a history x such that all nodes v_i have $R_i(x) \leq 0$. Then the average number of successful transmissions is an $O(c \cdot \beta)$-approximation of the optimum.

Let I^* denote a maximum independent set. For $v_i \in I^*$, let $t_i = \sum_{t=1}^{T} x_i^t$ be the number of attempts by node v_i.

b) First, consider the case that at least half the nodes $v_i \in I^*$ have $t_i \geq T/2$. Prove the theorem for this case.

c) Second, take a look at the remaining case that at least half the nodes $v_i \in I^*$ have $t_i < T/2$. Prove the theorem for this case.