Theory of Distributed Systems

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Exercise 4

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Exercise 4.1. *ThreeColor2Matching on Trees* (4 Points)

Given a legal 3-coloring on a rooted tree, show that it is possible to find a maximal matching in time $O(1)$.

Exercise 4.2. *MST on a Ring* (4 Points)

Prove Lemma 34 from the notes:

Every distributed algorithm to compute an MST on the ring requires $\Omega(n)$ many rounds.

Exercise 4.3. *Blue-Red Edges* (6 Points)

Consider a weighted graph $G = (V, E, \omega)$ with distinct edge weights. Recall the following from the lecture:

- An edge e is a red edge if there is a cycle in G and e has the highest weight on that cycle.
- An edge e is a blue edge if there is a fragment of the MST T^* such that e has minimum weight among all outgoing edges of the fragment.

Show that there cannot be a single edge that is both red and blue.

Exercise 4.4. *Adaptation of Dual Greedy* (5 Points)

Consider the following variant of the Dual Greedy algorithm where the repeat-loop (lines 3-11 in the notes) is changed:

Instead of waiting for a message from every child, each node v starts sending non-eliminated edges to its parent node immediately. The node continues to send until the end of round $\hat{L}(v) + n$. If there is no eligible unsent edge, the node waits for the next round.

Show that by round t, all edges sent from a node v to its parent u in the original Dual Greedy algorithm have also been sent from v to u in this variant.

Exercise 4.5. *Bonus* (6* Points)

Show that there is a distributed algorithm to construct an MST on a complete graph with n vertices in $O(\log n)$ rounds.
Note that the nodes are aware of the fact that they are in a complete graph.

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