

Exercise 3

Issued: 06.11.2018

Due: 13.11.2018

Update 2018-11-08: Task 3.2b) has been updated

Presenting good solutions during the exercise sessions will be rewarded with extra points.

Exercise 3.1. Ring-3-Coloring

(4 Points)

Given an MIS on a ring with IDs and without consistent orientation, prove that it is possible to deterministically 3-color the vertices in a single round.

Exercise 3.2. Anonymous rings

(4 + 4 = 8 Points)

Consider an anonymous ring (without IDs, every node has the same input) with simultaneous wakeup.

- Suppose the ring has no consistent orientation. Given an MIS, prove that it is impossible to deterministically 3-color the vertices.
- Show Lemma 22 in the notes:
Prove that it is impossible to compute an MIS deterministically even if the ring has a consistent orientation.

Exercise 3.3. MIS-Rank

(4 + 3 = 7 Points)

Consider the MIS-Rank algorithm from the lecture. Let G be a graph.

- Show that the bound on the time complexity cannot be improved to $O(\text{Diam}(G))$.
- A *maximum* independent set M of G is an MIS with highest cardinality among all MIS of G : For all MIS M' of G it holds that $|M| \geq |M'|$.
Give an example of a graph G and an ID-assignment on which MIS-Rank does not compute a maximum independent set.

Exercise 3.4. Bonus

(6* Points)

Show Theorem 13 in the notes:

Show that an algorithm computing an MIS on an arbitrary graph G in time $T(G)$ in the LOCAL model can be used to compute a $(\Delta + 1)$ -coloring of G in time $O(T(G))$.