## Theory of Distributed Systems

Winter Term 2018/2019

Prof. Dr. Martin Hoefer, Niklas Hahn

Exercise 3

Update 2018-11-08: Task 3.2b) has been updated Presenting good solutions during the exercise sessions will be rewarded with extra points.

## Exercise 3.1. Ring-3-Coloring

Given an MIS on a ring with IDs and without consistent orientation, prove that it is possible to deterministically 3-color the vertices in a single round.

**Exercise 3.2.** Anonymous rings

Consider an anonymous ring (without IDs, every node has the same input) with simultaneous wakeup.

- a) Suppose the ring has no consistent orientation. Given an MIS, prove that it is impossible to deterministically 3-color the vertices.
- b) Show Lemma 22 in the notes: Prove that it is impossible to compute an MIS deterministically even if the ring has a consistent orientation.

Exercise 3.3. MIS-Rank

Consider the MIS-Rank algorithm from the lecture. Let G be a graph.

- a) Show that the bound on the time complexity cannot be improved to O(Diam(G)).
- b) A maximum independent set M of G is an MIS with highest cardinality among all MIS of G: For all MIS M' of G it holds that  $|M| \ge |M'|$ .

Give an example of a graph G and an ID-assignment on which MIS-Rank does not compute a maximum independent set.

## Exercise 3.4. Bonus

Show Theorem 13 in the notes:

Show that an algorithm computing an MIS on an arbitrary graph G in time T(G) in the LOCAL model can be used to compute a  $(\Delta + 1)$ -coloring of G in time O(T(G)).

The assignments and further information concerning the lecture can be found at http://algo.cs.uni-frankfurt.de/lehre/tds/winter1819/tds1819.shtml

UNIVERSITÄT FRANKFURT AM MAIN

JOHANN WOLFGANG

Institute of Computer Science Algorithms und Complexity

> Issued: 06.11.2018 Due: 13.11.2018

> > (4 Points)

(4 + 3 = 7 Points)



(4 + 4 = 8 Points)

$$(6* \text{ Points})$$