Theory of Distributed Systems

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Exercise 2

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Exercise 2.1. *Individual Messages* (6 + 2 = 8 Points)

Consider an n-vertex network $G = (V, E), V = \{v_1, \ldots, v_n\}$. In the following, let D denote the diameter of the graph G. The **individual messages** (IM) cast requires vertex v_1 to deliver a (distinct) $(\log n)$-bit message to every other vertex in the network along some prespecified shortest route.

a) Consider the synchronous $\mathcal{CONGEST}$-model. Prove or disprove each of the following claims regarding the time complexity of the problem.

   i Time(IM) $= O(D)$ (i.e. there exists a constant $c > 0$ such that for every network G as above, Time(IM,G) $\leq cD$).

   ii Time(IM) $= O(n)$ (i.e. there exists a constant $c > 0$ such that for every network G as above, Time(IM,G) $\leq cn$).

   iii Time(IM) $= \Omega(n)$ (i.e. there exists a constant $c > 0$ such that for every $n \geq 1$, there exists an n-vertex network G as above for which Time(IM,G) $\geq cn$).

   iv There exists a constant $c > 0$ such that for every network G as above, Time(IM,G) $\geq cD$.

b) Now, consider the synchronous $\mathcal{LOCAL}$-model. Prove or disprove Time(IM) $= O(D)$ (i.e. there exists a constant $c > 0$ such that for every network G as above, Time(IM,G) $\leq cD$).

Exercise 2.2. *Smallest k of m* (2 + 3 = 5 Points)

The following policy is proposed for solving the "smallest k of m" problem described in section 4.3.1 of the Peleg book:

- At any given moment along the execution, every vertex keeps only the set of (at most) k smallest items it knows.
- In each step, each vertex sends to its parent an *arbitrary* element from this set that has not been sent yet.

a) Prove that using this policy, the k smallest elements must still arrive at the root by time $O(k \cdot Depth(T))$.

b) For every integer $m \geq 1$, give an example of a tree T and an initial distribution of m elements, where $k = Depth(T) = \lceil \sqrt{m} \rceil$ and the above process takes $\Omega(m)$ steps.

Please turn over!

Exercise 2.3. *Bellman-Ford* (5 Points)

Modify the Bellman-Ford algorithm so that it detects its termination. Make sure that the given bounds on the time and message complexity still hold.

Exercise 2.4. *Coloring* (3 + 3 = 6 Points)

Consider the expansion of the tree-coloring algorithm from the lecture for arbitrary, bounded degree graphs. Here, we just concatenate for all $w \in \Gamma(v), w \neq v$ the strings which the original algorithm would have chosen as new color of node v under the assumption that neighbor w was v's parent in the tree. This reduction step, starting with a legal K-bit coloring, yields a $\Delta(\lceil \log K \rceil + 1)$-bit coloring. Assume that the resulting coloring has exactly this length, meaning that each of the entrys that could possibly be shorter is filled with leading zeros.

a) Prove that the coloring obtained from this reduction step is legal.

b) Show that the minimum number of bits that can be achieved by iterating this reduction step is of order $\Theta(\Delta \log \Delta)$, i.e. prove corresponding upper and lower bounds on K in $O()$- and $\Omega()$-notation.

This result is given as Theorem 7.3.8 in Peleg's book. Note that Peleg's estimate of 2Δ instead of $O(\Delta \log \Delta)$ is a bit too optimistic.

The assignments and further information concerning the lecture can be found at
`http://algo.cs.uni-frankfurt.de/lehre/tds/winter1819/tds1819.shtml`

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