Theory of Distributed Systems

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Exercise 9

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Exercise 9.1. *ALOHA* (4 Points)

Show Lemma 61 from the notes:

The Slotted ALOHA protocol elects a leader in $O(\log n)$ rounds w.h.p.

Exercise 9.2. *k-Channel Leader Election* ($3 + 4 = 7$ Points)

Consider a wireless network as a clique of n nodes. There are $k = \sqrt{n}$ independent channels numbered from 1 to $\sqrt{n}$. In each round t, each node j chooses one channel k_j^t. Then j can transmit or listen in round t only on k_j^t. Transmissions of two or more nodes on distinct channels do not interfere, on the same channel they yield a collision. Nodes do not have collision detection.

Suppose we want to elect a unique leader for each channel.

a) Prove that the following algorithm needs $O(\sqrt{n}\log n)$ time steps whp:

   In phase $i = 1 \dots \sqrt{n}$, every remaining device runs the Slotted ALOHA protocol on channel i until a leader for channel i is elected. The leader for channel i gets informed and drops out. The transmitting probability is always fixed to $1/n$.

b) Consider the following algorithm:

   In the beginning, each node selects a single channel uniformly at random. Then each node runs Slotted ALOHA only on its selected channel. If in the end there is a channel without leader, the algorithm restarts. The transmitting probability is always fixed to $1/\sqrt{n}$.

   Show that the algorithm needs $O(\log n)$ time steps w.h.p.

Exercise 9.3. *Max-Average-Degree* (4 Points)

Give an example of a graph G with n nodes with $MaxAvg(G) = \Theta(n)$ and $\chi(G) = O(1)$. Here, $\chi(G)$ is the chromatic number of the graph.

Please turn over!

Exercise 9.4. *Disk-Graphs on the Line* (3 + 3 = 6 Points)

Suppose there are n base stations located along a line. Each base station tries to reach mobile receivers in the vicinity on the line.

Formally, we assume for each base station i there is a continuous line segment of length $\ell_i > 0$ and the base station is located in the middle of this segment. Two base stations are conflict-free if and only if their segments do not intersect.

The resulting conflict graph G can be seen as a "one-dimensional disk-graph". Show the following bounds on the inductive independence number $\rho(G)$:

a) $\rho(G) \leq 2$

b) $\rho(G) \leq 1$

Hint: Obviously, a solution to b) is sufficient to solve a) as well.

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