Theory of Distributed Systems

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Exercise 6

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This is the last exercise sheet for Part 1 of the lecture.

For all tasks we consider the synchronous CONGEST-model. Further, we consider connected, weighted graphs $G = (V, E, \omega)$. Note that the weights do not have to be distinct.

Consider the following variant of the Mailing Problem, the **Orthogonal Mailing Problem**:

Given a graph G with two specified nodes $s \neq r$ as well as bit-vectors $b^{(s)}$ and $b^{(r)}$ of size k for s and r, respectively. Find out whether the bit vectors are orthogonal, i.e., r wants to find out if $\sum_{i=1}^{k} b_i^{(s)} b_i^{(r)} = 0$.

Lemma:
For every $m \geq 1$, the Orthogonal Mailing Problem for $k = m^2$ cannot be solved in time $o(m^2/\log m)$ on the hard graph HG_m by a distributed algorithm.

Exercise 6.1. *Weighted Distances* (6 Points)

Use the above lemma to show that in the class of hard graphs finding any approximation to the weighted distance between a pair of nodes $s, t \in V$ takes $\Omega(\sqrt{n}/\log n)$ rounds.

Exercise 6.2. *Weighted Cuts* (6 Points)

For $s, t \in V$, an s-t-cut is a subset $S \subseteq V$ with $s \in S$ and $t \notin S$. The weight of the cut is the sum of the weights of all edges $\{v, w\} \in E \cap (S \times (V \setminus S))$, i.e. the edges crossing the cut.

Use the above lemma to show that in the class of hard graphs finding any approximation to the weight of a minimum s-t-cut takes $\Omega(\sqrt{n}/\log n)$ rounds.

Exercise 6.3. *MSTs on Rings* (8 = 4 + 4 Points)

Consider an n-vertex ring with weighted edges. Note that the weights do not have to be distinct.

a) Assume there is a root node given. Provide a deterministic distributed algorithm for MST construction. By the end of the algorithm, each vertex should know its parent and its child in the MST. The algorithm should take at most n (not $O(n)$) many time steps.

b) Prove Lemma 34 from the notes:

   Every distributed algorithm to compute an MST on the ring requires $\Omega(n)$ many rounds.