Theory of Distributed Systems

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Exercise 4

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Exercise 4.1. *Ring-3-Coloring* (4 Points)

Given an MIS on a ring with IDs and without consistent orientation, prove that it is possible to deterministically 3-color the vertices after a single round with communication.

Exercise 4.2. *Anonymous rings* (8 = 4 + 4 Points)

Consider an anonymous ring (without IDs, every node has the same input) with simultaneous wakeup.

a) Suppose the ring has no consistent orientation. Given an MIS, prove that it is impossible to deterministically 3-color the vertices.

b) Show Lemma 22 in the notes:
Prove that it is impossible to compute an MIS deterministically even if the ring has a consistent orientation.

Exercise 4.3. *MIS-Rank* (8 = 4 + 4 Points)

Consider the MIS-Rank algorithm from the lecture. Let G be a graph.

a) Show that the bound on the time complexity cannot be improved to $O(Diam(G))$.

b) Now consider a path graph G with IDs taken to be a random permutation of $\{1, \ldots, n\}$. Prove that in this case

$$\text{Time(MIS-Rank,G)} > c \log_2 n / \log_2(\log_2 n)$$

for a constant $c > 0$ with probability at most $1/n$.

Hint: The inequality $k! \geq (k/2)^{k/2}$ for each $k \in \mathbb{N}$ may help.

Exercise 4.4. *Bonus* (6* Points)

Show Theorem 13 in the notes:
Show that an algorithm computing an MIS on an arbitrary graph G in time $T(G)$ in the LOCAL model can be used to compute a $(\Delta + 1)$-coloring of G in time $O(T(G))$.

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