### 1

# **Optimization and Uncertainty**

Summer term 2023

Assignment 8

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Exercise 8.1 Online PERSUADE

Consider n boxes and assume that all boxes and distributions  $\mathcal{D}_i$ , i = 1, ..., n, are known. The boxes are opened in a known order. In round i, S opens box i and sends a signal to  $\mathcal{R}$  ("take box i" or "do not take box  $i^{"}$ ). If  $\mathcal{R}$  does not take the box, the process continues with the next box. Otherwise, if  $\mathcal{R}$  takes the box, the process stops. Note that when sending the signal in round *i*,  $\mathcal{S}$  only knows the content of boxes 1, ..., i.

- a) For IID boxes, show that there exists a direct and persuasive  $(1-1/e)^{-1}$ -competitive algorithm. Hint: Use a modification of Algorithm 12 from the lecture notes where, instead of a random box, the modified algorithm takes either the first yes-box or the last no-box (the rest remains unchanged).
- b) For independent (but not necessarily identical) boxes and the SSQ condition fulfilled with the SSQ box in round  $i^* = n$ , show that there exists an algorithm with constant competitive ratio.
- c) For independent (but not necessarily identical) boxes and the SSQ condition fulfilled with the SSQ box in round  $i^* < n$ , show that there is no algorithm with finite competitive ratio.

## **Exercise 8.2** DELEGATION

A class of instances for DELEGATION with  $n \ge 2$  IID boxes is given as follows:

For each box  $i \in [n]$ , a random prize-pair  $(s_i, r_i) \in \{0, 1\}^2$  is obtained with two independent Bernoulli trials such that  $\Pr[s_i = 1] = \Pr[r_i = 1] = \frac{1}{n}$  and  $\Pr[s_i = 0] = \Pr[r_i = 0] = 1 - \frac{1}{n}$ .

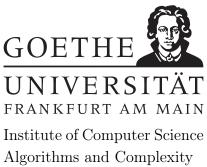
Assume that  $\mathcal{S}$  breaks ties in favor of  $\mathcal{R}$ .

- a) Apply Algorithm 14 from the lecture on this class of instances. Show your calculations for each step of the algorithm and write down the distribution of the computed decision scheme  $\psi$ .
- b) Calculate the expected reward for  $\mathcal{R}$  when using Algorithm 14 for computing a decision scheme.
- c) Find a best possible decision scheme for  $\mathcal{R}$  for this class of instances or argue why Algorithm 14 yields a best possible decision scheme here.

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(2 + 1 + 1 = 4 points)



(4 + 2 + 4 = 10 points)

### Exercise 8.3 Online DELEGATION

(1 + 2 + 3 = 6 points)

Consider an online variant of DELEGATION where n independent boxes arrive sequentially in a fixed order. Both the sender S and the receiver  $\mathcal{R}$  know the order of the boxes and their respective distributions  $\mathcal{D}_1, \ldots, \mathcal{D}_n$  in advance, and  $\mathcal{R}$  commits to a decision scheme  $\psi$ . At arrival of box i, S looks in the box (i.e., sees prize-pair  $\theta_{ij} = (s_{ij}, r_{ij})$  randomly drawn according to distribution  $\mathcal{D}_i$ ) and decides immediately whether to recommend it to  $\mathcal{R}$  or not. If S lets it pass (and i < n), the next box i + 1 arrives. The process ends when S recommends a box (upon which  $\mathcal{R}$  makes the accept/reject decision according to  $\psi$ ) or if S has let all n boxes pass.

The approximation factor of a decision scheme for this problem is defined with respect to the expected optimal prize that  $\mathcal{R}$  could get in an online scenario without delegation on the same instance.

a) Consider the following decision scheme  $\psi$ : Choose a single box  $i^* := \arg \max_{i \in [n]} \mathbb{E}[r_{ij}]$  from which the maximum expected prize for  $\mathcal{R}$  is obtained. Always accept any recommendation from that box  $i^*$  and reject everything else, i.e.,  $\psi(\theta_{i^*j}) = 1$  for all  $j \in m_{i^*}$  and  $\psi(\theta_{ij}) = 0$  for all  $i \neq i^*, j \in m_i$ .

Show that this decision scheme is always n-approximative.

- b) Consider an instance of Online DELEGATION with some box i < n and prize-pair  $\theta_{ij}$ . Moreover, there is another box i' > i with a prize-pair  $\theta_{i'j'}$  such that  $s_{i',j'} \ge n^2 \cdot s_{ij}$  and  $\Pr[\theta_{i'j'}] \ge \frac{1}{n}$ . Prove: For any decision scheme  $\psi$  with  $\psi(\theta_{i'j'}) = 1$ , it holds that S never recommends  $\theta_{ij}$ .
- c) Construct a class of instances of Online DELEGATION where every decision scheme is Ω(n)-approximative. Show the correctness of your construction.
  *Hint: Use insights from b*).

The assignments and further information on the course are provided on our website: https://algo.cs.uni-frankfurt.de/lehre/oau/sommer23/oau23.shtml

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