Optimization and Uncertainty

Summer term 2021

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Assignment 7

Issued: 08.06.2021
Due: 22.06.2021, **10:00h**

The handling time of this assignment is **two weeks**. It will be discussed in the exercise session on 25.06.2021. There will be **no exercise session on 18.06.2021**.

This is the first assignment counting for the **second part** of the lecture.

Exercise 7.1 *Direct and persuasive schemes* (3 points)

Show Proposition 3 from the lecture notes: *For every signaling scheme φ there is a direct and persuasive scheme φ' such that $\mathcal{S}$ obtains the same expected value in φ and φ'.*

Exercise 7.2 Random-Order Signaling (3 + 4 + 4 points)

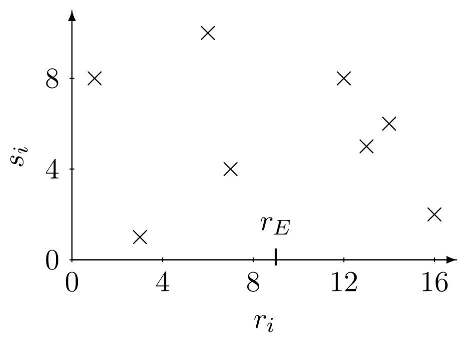
In Random-Order Signaling, n prize-pairs are revealed to both the sender $\mathcal{S}$ and the receiver $\mathcal{R}$ at the beginning. Afterwards, the prize-pairs are packed into n boxes. The boxes are closed (which makes them look identical), permuted in uniform random order, and then labeled from 1 to n. Now $\mathcal{S}$ can look into all boxes and then sends a signal to $\mathcal{R}$.

a) Show that the optimal scheme φ^* for $\mathcal{S}$ is symmetric.

b) Describe an algorithm to compute φ^* in polynomial time. Give an informal argument why your algorithm yields an optimal policy indeed.
Hint: For starters, consider the geometric visualization of the example instance below. The instance consists of eight boxes with prize-pairs (marked by crosses) as shown in the table. As introduced in the lecture, r_i and s_i denote the prizes of $\mathcal{R}$ and $\mathcal{S}$ for box i, respectively. Here, r_E denotes the expected prize of $\mathcal{R}$. What are the relevant boxes for $\mathcal{S}$? Which of these boxes is $\mathcal{R}$ willing to accept? What is the optimal weighting of the signals for $\mathcal{S}$?

c) Assume $\mathcal{S}$ could choose the best box i for her by herself (instead of sending a signal to $\mathcal{R}$). Show that the ratio of the expected rewards of i and φ^* for $\mathcal{S}$ can be up to n, and this is the worst case.

	1	2	3	4	5	6	7	8
r_i	1	3	6	7	12	13	14	16
s_i	8	1	10	4	8	5	6	2



Exercise 7.3 *Online* PERSUADE (4 + 1 + 4 + 4 points)

Consider n boxes and assume that all boxes and distributions $\mathcal{D}_i$, $i = 1, ..., n$, are known. The boxes are opened in a known order. In round i, $\mathcal{S}$ opens box i and sends a signal to $\mathcal{R}$ ("take box i" or "do not take box i"). If $\mathcal{R}$ does not take the box, the process continues with the next box. Otherwise, if $\mathcal{R}$ takes the box, the process stops. Note that when sending the signal in round i, $\mathcal{S}$ only knows the content of boxes $1, ..., i$.

a) For IID boxes, show that there exists a direct and persuasive $(1-1/e)^{-1}$-competitive algorithm.
Hint: Use a modification of Algorithm 12 from the lecture notes where, instead of a random box, the modified algorithm takes either the first yes-box or the last no-box (the rest remains unchanged).

b) For independent (but not necessarily identical) boxes and the SSQ condition fulfilled with the SSQ box in round $i^* = n$, show that there exists an algorithm with constant competitive ratio.

c) For independent (but not necessarily identical) boxes and the SSQ condition fulfilled with the SSQ box in round $i^* < n$, show that there is no algorithm with finite competitive ratio.

d) There exists an algorithm Γ which computes the optimal signaling scheme in polynomial time for the independent case (even if the SSQ condition does not hold). It operates via backwards induction and solves $n-1$ linear programs. More precisely, Γ uses a LP to compute the optimal mechanism in each round $i = 1, .., n$.

   Depict the LP for round i. Show that it always has a feasible solution.
   Hint: Suppose we have computed the optimal mechanism to be applied in rounds $i + 1, ..., n$. Now consider round i. Under what circumstances does $\mathcal{S}$ send "take box i" to $\mathcal{R}$? In what cases would $\mathcal{R}$ take the recommended box indeed? Bear in mind that the mechanism signals at most one "take it"-signal in total since the scheme is supposed to be persuasive.

The assignments and further information on the course are provided on our website:
http://algo.cs.uni-frankfurt.de/lehre/oau/sommer21/oau21.shtml

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