Algorithmic Game Theory

Winter Term 2023/2024

Prof. Dr. Martin Hoefer
Marco Schmalhofer, Lisa Wilhelmi

GOETHE UNIVERSITÄT FRANKFURT AM MAIN
Institute for Computer Science
Algorithms and Complexity

Assignment 11

Issued: Jan 30, 2024
Due: Feb 5, 2024, **23:55h**

This is the last assignment of part II and be discussed in the exercise session on Feb 9.

Exercise 11.1. (2 + 2 + 2 + 2 + 2 Points)

Consider a beach that can be represented by the interval $[0, 1]$. There are n people visiting the beach and visitor i has a most favorite spot $s_i \in [0, 1]$. We would like to place ice-cream sellers at the beach. We ask every visitor for the most preferred position $b_i \in [0, 1]$ and each visitor i has an incentive that an ice-cream seller gets placed as close as possible to s_i. Let $b = (b_1, \dots, b_n)$, and $\mathcal{N} = \{1, \dots, n\}$.

First, assume that we only place a single ice-cream seller at position $p_1 \in [0, 1]$.

a) Let $d^{\Sigma}(p_1, b)$ be the total distance of all reported positions to the ice-cream seller at p_1, i.e.,

$$d^{\Sigma}(p_1, b) = \sum_{i=1}^{n} |b_i - p_1| \ .$$

Prove or disprove: There is an incentive-compatible mechanism without money such that $d^{\Sigma}(p_1, b)$ is minimized.

b) Consider the maximum distance of any visitor to p_1, i.e.,

$$d^{\max}(p_1, b) = \max_{i \in \mathcal{N}} |b_i - p_1| \ .$$

Prove or disprove: There is an incentive-compatible mechanism without money such that $d^{\max}(p_1, b)$ is minimized.

For the following tasks, assume we place two ice-cream sellers at $p_1, p_2 \in [0, 1]$.

c) Consider again the maximum distance of any visitor to the next ice-cream seller. Let

$$d^{\max}(p_1, p_2, b) = \max_{i \in \mathcal{N}} \{\min(|b_i - p_1|, |b_i - p_2|)\} \ .$$

Prove or disprove: There is an incentive-compatible mechanism without money such that $d^{\max}(p_1, p_2, b)$ is minimized.

d) Consider the following max-min-mechanism: Choose $p_1 = \min_{i \in \mathcal{N}} b_i$ and $p_2 = \max_{i \in \mathcal{N}} b_i$. Prove or disprove: This mechanism is incentive compatible.

e) Prove that the max-min-mechanism is a 2-approximation for the maximum distance, i.e.,

$$d^{\max}(p_1, p_2, b) \quad \leq \quad 2 \cdot \min_{q_1, q_2 \in [0,1]} d^{\max}(q_1, q_2, b) \ .$$

Exercise 11.2. (2 + 2 + 2 Points)

Prove the following statements:

a) The ~~Random~~ Serial Dictatorship (RSD) algorithm is incentive compatible for every a-priori fixed permutation π of players.

b) There is an instance and a permutation such that the outcome of the RSD algorithm is not in the core.

c) For every instance there is a permutation such that the outcome of the RSD is in the core.

Exercise 11.3. (4 Points)

Prove that the matching mechanism with priority lists for kidney exchange is incentive compatible. Here, we assume that players are the patient-donor pairs. It is sufficient to show that an unmatched player, i.e., a patient-donor pair, cannot get included into the matching by not reporting all their compatibilities.

Exercise 11.4. (4 Points)

In the TTC mechanism initially player i owns house i. Consider a group $S \subseteq \{1, \ldots, n\}$ of players trying to cheat in the following way: The agents in S permute their houses before entering the mechanism. However, they reveal their preferences over houses truthfully.

Is there an instance where it is possible for a group S to improve at least one player in S by cheating in the above way while no other player of S gets worse? Prove your answer.

Contact: Marco Schmalhofer, Lisa Wilhelmi (`[lastname]@em.uni-frankfurt.de`).