Algorithmic Game Theory

Winter Term 2023/2024

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Assignment 7

Issued: Dec 19, 2023
Due: Jan 8, 2024, **23:55h**

This is the first assignment of part II and will be discussed in the exercise session on Jan 12.

Exercise 7.1. (3 + 3 Points)

There is an auction with one good and $n > 2$ bidders with private valuations for the good. We assume that their bids are pairwise distinct. The good is assigned to the bidder with the highest bid, and the price is p. The other bidders neither receive nor pay anything.

Is the described mechanism incentive compatible, when p is

a) the arithmetic mean of all bids,

b) the third highest bid?

Prove your answers.

Exercise 7.2. (6 Points)

We consider a combinatorial auction: There is a set $\mathcal{G}$ of goods that are indivisible. Bidders have a private valuation for each subset of $\mathcal{G}$. Every outcome allocates exactly one subset of $\mathcal{G}$ to every bidder such that the intersection of each pair of allocated subsets is empty.

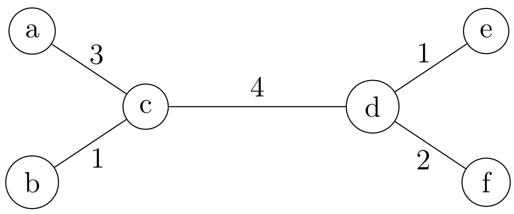
Let $\mathcal{G} = \{A, B, C\}$ and $\mathcal{N} = \{1, 2, 3, 4\}$. The private valuations $v_i(G)$ for all bidders $i \in \mathcal{N}$ and subsets $G \subseteq \mathcal{G}$ are given by the following table:

	$\emptyset$	$\{A\}$	$\{B\}$	$\{C\}$	$\{A, B\}$	$\{A, C\}$	$\{B, C\}$	$\{A, B, C\}$
$i = 1$	0	2	2	3	4	2	6	6
$i = 2$	0	0	3	0	6	0	5	9
$i = 3$	0	1	3	2	5	3	3	5
$i = 4$	0	1	1	5	1	4	1	2

Apply the VCG mechanism with Clarke rule on this instance. Explain which allocation is chosen by the mechanism, and why. Then calculate the payments for all bidders (with explanation). You may assume truthful bidding.

Exercise 7.3. (4 + 3 Points)

In a *path auction* there is an underlying network given as an undirected graph $G = (V, E)$ with edge capacities $c_e \in \mathbb{N}$ for all $e \in E$. Every bidder $i \in N$ has a desired path $P_i \subseteq E$, a demand $d_i \in \mathbb{N}$ and a valuation $v_i \in \mathbb{R}_{\geq 0}$ for getting allocated. Every bidder either gets allocated her path (valuation v_i) or nothing (valuation 0). Based on the bids of the players, the auctioneer decides which player is allowed to use her desired path and for which price. However, the allocation of paths to bidders needs to be feasible in the following sense: The sum of demands of players using an edge e may not exceed the capacity c_e. We do not allow fractional assignments.



Bidder	Path	Demand d_i	Value v_i
1	{ac, cd}	2	12
2	{bc, cd, de}	1	6
3	{ed, df}	1	3
4	{cd, df}	3	10

a) Consider the following allocation rule:

- Order the bidders by their bid per unit of demand $\frac{b_i}{d_i}$ in non-increasing order. Break ties arbitrarily.
- Iterate over the ordered bidders and greedily allocate their path if possible.

Can this allocation rule be used in an incentive-compatible mechanism for path auctions? Prove your answer. If it is 'no', give an IC mechanism for path auctions.

b) Apply the mechanism of a) to the given example assuming truthful bidding. Compute the outcome and the prices for all bidders. Does the allocation rule always maximize social welfare?

Exercise 7.4. (4 Points)

In this exercise we want to show a (weak) Roberts-like theorem. Assume there are only $|A| = 2$ outcomes and $n = 2$ players with arbitrary valuations $V_i = \mathbb{R}^A$. Consider the allocation rule

$$f(v_1, \ldots, v_n) \in \arg\max_{a \in A} \sum_{i=1}^{n} (v_i(a))^2 .$$

Prove that for this rule there are no payments $p_1, \ldots, p_n$ such that the mechanism becomes incentive compatible.

Hint: Find concrete valuations (v_1, v_2) and (v_1', v_2) such that using the direct characterization of IC for player 1 allows to derive conditions for the payments that yield a contradiction.

Assignments and further information concerning the course can be found at
`https://algo.cs.uni-frankfurt.de/lehre/agt/winter2324/agt2324.shtml`

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