Algorithmic Game Theory

Winter Term 2023/2024

Prof. Dr. Martin Hoefer
Marco Schmalhofer, Lisa Wilhelmi

GOETHE UNIVERSITÄT FRANKFURT AM MAIN
Institute for Computer Science
Algorithms and Complexity

Assignment 5

Issued: Nov 21, 2023
Due: Nov 27, 2023, **23:55h**

This assignment will be discussed in the exercise session on Dec 1st.

Exercise 5.1. (1 + 3 + 3 Points)

The following (4×4) bimatrix denotes **utilities** for both players in all states in a stategic game:

	E	F	G	H
A	2, 2	3, 8	8, 3	2, 2
B	3, 8	1, 1	0, 0	8, 3
C	8, 3	0, 0	1, 1	3, 8
D	2, 2	8, 3	3, 8	2, 2

Give an example for each of the following equilibria concepts for this game:

a) a mixed Nash equilibrium.

b) a correlated equilibrium that is not a mixed Nash equilibrium.

c) a coarse-correlated equilibrium that is not a correlated equilibrium.

Exercise 5.2. (2 + 3 + 3 + 3 Points)

Let a finite game Γ with n agents be given. The set of strategies of agent i is denoted by S_i.

a) Prove: Every correlated equilibrium $\mathcal{V}$ is also a coarse-correlated equilibrium.

b) Assume Γ is a symmetric (2×2) bimatrix game, where both players have the same set of strategies and identical costs in corresponding states, i.e., for all $(x, y) \in S_1 \times S_2$, there is a valid state $(y, x) \in S_1 \times S_2$ and it holds $c_1(x, y) = c_2(y, x)$.

   Show that every coarse-correlated equilibrium $\mathcal{V}$ is also a correlated equilibrium.

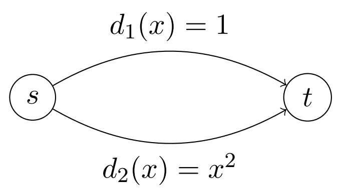
c) Show that every mixed Nash equilibrium in Γ is a coarse-correlated equilibrium in the following sense: Given a mixed Nash equilibrium $(x_{ij})_{i\in N, j\in S_i}$, we define the distribution over states $\mathcal{V}$ by

$$p(s) = \prod_{i\in N} x_{i,s_i} \ .$$

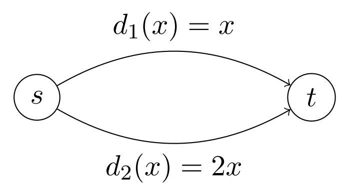
Prove that $\mathcal{V}$ is a coarse-correlated equilibrium.

d) Prove: If $\mathcal{V}'$ and $\mathcal{V}''$ are correlated equilibria then $\lambda\mathcal{V}' + (1-\lambda)\mathcal{V}''$ is also a correlated equilibrium, for $\lambda \in (0,1)$.

Exercise 5.3. (4 + 3 Punkte)



Network i)



Network ii)

a) Consider the two given instances of Wardrop games above. Derive an upper bound on the price of anarchy for both instances.

b) Let a Wardrop game on a directed graph G with nodes s and t be given, where there exists at least one path from s to t. For every edge $e \in E$, the latency function is given by $d_e(x) = a_e x^2 + b_e x + c_e$, where $a_e, b_e, c_e \geq 0$. Show that the price of anarchy is at most

$$\frac{1}{1 - \frac{2}{3\cdot\sqrt{3}}} \approx 1.625 \ .$$

Hint: Consider the proof regarding affine latency functions from the lecture.