Algorithmic Game Theory

Winter Term 2022/2023

Prof. Dr. Martin Hoefer, Dr. Giovanna Varricchio
Conrad Schecker

GOETHE UNIVERSITÄT FRANKFURT AM MAIN
Institute for Computer Science
Algorithms and Complexity

Assignment 11

Issued: Jan 24, 2023
Due: Jan 31, 2023, **10:00h**

Exercise 11.1. (2 + 2 Points)

Prove the following statements:

a) The ~~Random~~ Serial Dictatorship (RSD) algorithm is incentive compatible for every a-priori fixed permutation π of players.

b) There is an instance and a permutation such that the outcome of the RSD algorithm is not in the core.

Exercise 11.2. (4 Points)

Prove that the matching mechanism with priority lists for kidney exchange is incentive compatible. Here, we assume that players are the patient-donor pairs. It is sufficient to show that an unmatched player cannot get included into the matching by not reporting a compatibility.

Exercise 11.3. (3 Points)

We consider a variant of the kidney exchange problem with hospitals. There is a set H of hospitals and a set V of n patient-donor pairs. Each patient-donor pair $v \in V$ is located at some hospital $h(v)$. Every hospital $h \in H$ reports the local patient-donor pairs $V(h) = \{v \in V : h(v) = h\}$ to the national kidney exchange. The central exchange finds compatibilities, i.e., a set E of edges such that the graph $G = (V, E)$ connects all compatible patient-donor pairs.

Any mechanism for this problem applies a matching procedure on G and assigns the matches to hospitals. The objective is to find a matching that maximizes the total number of matched patient-donor pairs. A match $\{u, v\}$ can be assigned to either $h(u)$ or $h(v)$. For simplicity, assume that if $h(u) \neq h(v)$, both hospitals have a probability of $\frac{1}{2}$ for receiving the match.

Hospitals have a utility of $k > 0$ for every received match and want to maximize their (expected) utility. They can neither report non-existing pairs nor false information affecting compatibility. However, they might report only a subset of $V(h)$: It is possible for hospitals to determine the compatibility of local patient-donor pairs and match them internally without reporting. Their utility is still k for such match.

Prove that there is no incentive compatible mechanism that guarantees to match the maximum number of possible patient-donor pairs.

Exercise 11.4. (1 + 2 Points)

Consider the MovingKnife protocol and let $\pi : \mathcal{N} \to \mathcal{N}$ be the permutation that describes the order of assignments, i.e. agent i is the $\pi(i)$-th player to receive a piece of cake.

a) Prove: For each pair of agents $i, j \in \mathcal{N}$ with $\pi(i) < \pi(j)$, it holds that j never envies i.

b) Find an instance with a minimum number of agents that fulfills the following requirement: There is a pair of agents $j, k \in \mathcal{N}$ with $\pi(j) < \pi(k)$ such that j envies k.

   Prove your answer.

Exercise 11.5. (2 + 1 + 1 Points)

Consider the CutAndChoose protocol.

a) Is the protocol incentive compatible?

b) Explain how to implement the protocol with minimum query complexity when only the $Cut_i(x, \alpha)$ query is available.

c) Explain how to implement the protocol with minimum query complexity when only the $Eval_i(x, y)$ query is available.

Prove your answers.