Algorithmic Game Theory

Winter Term 2022/2023

Prof. Dr. Martin Hoefer, Dr. Giovanna Varricchio
Conrad Schecker

GOETHE UNIVERSITÄT FRANKFURT AM MAIN
Institute for Computer Science
Algorithms and Complexity

Assignment 6

Issued: Nov 29, 2022
Due: Dec 06, 2022, **10:00h**

Exercise 6.1. (1 + 4 + 3 Points)

The following (3×3) bimatrix denotes **utilities** for both players in all states in a stategic game:

	E	F	G
A	0 1	0 0	1 0
B	1 0	0 1	0 0
C	0 0	1 0	0 1

Give an example for each of the following equilibria concepts for this game:

a) a mixed Nash equilibrium.

b) a correlated equilibrium that is not a mixed Nash equilibrium.

c) a coarse-correlated equilibrium that is not a correlated equilibrium.

Exercise 6.2. (1 + 4 Points)

Consider a finite (cost minimization) game Γ with the set $\mathsf{PNE}(\Gamma)$ of pure Nash equilibria, and let $\mathcal{V}$ be a distribution over states of Γ.

a) Suppose Γ has pure Nash equilibria and $\mathcal{V}$ is a distribution over them, i.e. $\mathsf{PNE}(\Gamma) \neq \emptyset$ and for every state $\tilde{s}$ with $\mathbb{P}_{s\sim\mathcal{V}}(s = \tilde{s}) > 0$, it holds $\tilde{s} \in \mathsf{PNE}(\Gamma)$.

   Prove: $\mathcal{V}$ is a correlated equilibrium for Γ.

b) Prove: If Γ is a symmetric (2×2) bimatrix game and $\mathcal{V}$ is a coarse-correlated equilibrium for Γ, then $\mathcal{V}$ is also a correlated equilibrium for Γ.

   Is this statement true even for asymmetric (2×2) bimatrix games?

   Remark: In a *symmetric* two-player game, both players have the same set of strategies and identical costs in corresponding states, i.e. for all $(x, y) \in S_1 \times S_2$, there is a valid state $(y, x) \in S_1 \times S_2$ and it holds $c_1(x, y) = c_2(y, x)$.

Exercise 6.3. (2 + 3 + 1 Points)

Consider the following modification of the game given in exercise 5.1:

	E	F	G
A	10 10	10 100	100 1
B	100 10	10 10	101 10
C	1 1	10 100	10 10

a) Prove that every finite sequence of states that contains the strictly dominated strategy G has a positive swap-regret.

b) Construct a sequence of states with the following properties:

- The no-swap-regret property is fulfilled for both players.
- The average strategy of the sequence doesn't converge to a mixed Nash equilibrium.
- The sequence does not contain strictly dominated strategies.

Prove the correctness of your construction.

Hint: Like in exercise 5.1, the sequence should be arbitrarily expandable: For any given $T' \in \mathbb{N}$, your sequence should be expandable to length $T \geq T'$.

c) Argue in at most two sentences how your sequence from b) could contain a strictly dominated strategy, but still fulfill the other two required properties.

Assignments and further information concerning the course can be found at
`http://algo.cs.uni-frankfurt.de/lehre/agt/winter2223/agt2223.shtml`

Contact: Conrad Schecker (`[lastname]@em.uni-frankfurt.de`).