# Algorithmic Game Theory

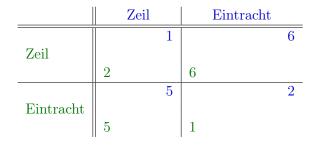
Winter Term 2022/2023

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## Assignment 3

### Exercise 3.1.

a) The bimatrix game *battle of sexes* is defined by the following matrix.



Construct an exact potential function for this game and prove its correctness.

b) Construct a  $2 \times 2$  bimatrix game with a pure Nash equilibrium and without exact potential function. Prove the correctness of your construction.

#### Exercise 3.2.

- a) Calculate all values of  $\alpha$  for which the function  $d_r : \{1, \ldots, n\} \to \mathbb{N}$  with  $d_r(x) = x^2$  is an  $\alpha$ -bounded jump.
- b) Let  $d : \{1, \ldots, n\} \to \mathbb{N}$  be a positive, monotonically increasing function with *bounded slope*, i.e. for any  $x_1, x_2 \in \{1, \ldots, n\}$  and some constant K, it holds:

$$|d(x_1) - d(x_2)| \le K \cdot |x_1 - x_2|.$$

Prove or disprove: d is a (K + 1)-bounded jump.

c) Let  $d: \{1, \ldots, n\} \to \mathbb{N}$  be a (K+1)-bounded jump, for some constant K. Prove or disprove: For all  $x_1, x_2 \in \{1, \ldots, n\}$ , it holds:

$$|d(x_1) - d(x_2)| \le K \cdot |x_1 - x_2|.$$



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(2 + 3 Points)

(2 + 2 + 2 Points)

#### Exercise 3.3.

Consider the game described in Exercise 2.4 from the previous exercise sheet:

There is a given graph G = (V, E) and given constants  $\kappa > 0$  and  $\alpha > 0$ . Each  $v \in V$  represents a player with strategy set  $\Sigma_v = \{0N, 0FF\}$  whose costs in any state S are given by

$$c_v(S) = \begin{cases} \kappa & , \text{ if } S_v = \texttt{ON}, \\ \alpha \cdot \left| \left\{ \left\{ u, v \right\} \in E \ : \ S_u = \texttt{OFF} \right\} \right| & , \text{ if } S_v = \texttt{OFF}. \end{cases}$$

a) Construct an isomorphic congestion game with strategy space  $\Sigma_i = \{ \mathsf{ON}_i, \mathsf{OFF}_i \}$  for all players  $i \in \mathcal{N}$ . Prove the correctness of your construction.

*Hint*: Define resources  $\mathcal{R}$ , delays  $(d_r(n_r))_{r \in \mathcal{R}}$  and the strategy space  $ON_i, OFF_i \subseteq \mathcal{R}$  for each  $i \in \mathcal{N}$  appropriately and show that every player has identical costs in corresponding states of both games.

b) Prove that every sequence of best response improvement steps has length  $\mathcal{O}(|V|^3)$ . What is the best upper bound that you can prove?