Algorithmic Game Theory

Winter Term 2022/2023

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Assignment 1

Issued: Oct 25, 2022
Due: Nov 01, 2022, **10:00h**

General Information on Submissions

Every submission for this course...

- ... must consist of a **single PDF file**.
- ... must be uploaded **before Tuesday, 10:00h** one week after the assignment was issued. You should have received a personal upload URL after signing up for the exercises.
- ... can be composed in English or German.
- ... will be graded based on correctness, completeness, comprehensibility and conciseness. In particular, **all answers require an explanation**, unless stated otherwise.

Exercise 1.1. (2 + 2 Points)

	E	F	G	H
A	8 7	7 4	2 9	6 4
B	4 6	8 3	1 7	5 2
C	9 6	2 6	5 8	2 3
D	4 8	6 5	4 7	9 9

Consider the 2-player game given by the matrix above. Calculate all

a) dominant strategies of the players,

b) pure Nash equilibria.

Exercise 1.2. (3 + 3 + 3 Points)

A strategy $s_i \in S_i$ of player i is called *strictly dominated* by strategy $s_i' \in S_i$, if s_i' is always strictly better then s_i, i.e. for all s_{-i} we have $c_i(s_i', s_{-i}) < c_i(s_i, s_{-i})$.

	W	X	Y	Z
A	5 3	5 9	2 6	4 5
B	2 6	5 5	3 9	6 9
C	1 4	9 7	5 2	6 4
D	1 4	7 4	7 8	2 8

a) Iteratively, eliminate all strictly dominated strategies in the cost matrix given above. Do this until there are no strictly dominated strategies left. Depict your cost matrix after each step.

b) Calculate a mixed Nash equilibrium for the reduced cost matrix.

c) Prove the following statement: In all 2-player normal form games, there is a one-to-one correspondence between mixed Nash equilibria in the original and the reduced game, when applying the reduction procedure described in a).

Exercise 1.3. (2 + 2 + 2 Points)

We generalize Sperner's Lemma to squares in the following way: We consider a square S that is subdivided into smaller squares by a grid of lines parallel to the edges of the original square. The vertices of the subdivision are the points of intersection of the lines. A *Sperner coloring* of S is a coloring of vertices that fulfills the following properties:

- The four outer corners of S are colored green, blue, orange, purple in clockwise order.
- Every vertex on the boundary (i.e. the outer side) of S is colored with one of the two colors of the endpoints of the corresponding outer line.
- Vertices in the interior of S are colored arbitrarily in one of the four colors.

An edge between a green and a blue vertex is called a *door*. Doors on the boundary of S are called *entrances*. Show the following properties for a Sperner coloring of a square S:

a) There is an odd number of entrances.

b) There is at least one small square with at least 3 different colors.

c) There is an odd number of small squares, with at least 3 different colors and exactly one door.

Assignments and further information concerning the course can be found at `http://algo.cs.uni-frankfurt.de/lehre/agt/winter2223/agt2223.shtml`

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