Algorithmic Game Theory

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Exercise Sheet 12

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Please hand in your solutions until Tuesday, February 11, 10:15h, in H9 or in the letterbox between rooms 114 and 115, R.M.S. 11-15.

Exercise 12.1. (4 Points)

Prove the Independence Lemma of the Online VCG Mechanism for a single item, i.e., show that for any round $t = 1, \dots, n$, for any two subsets $I_t, I_t^{'} \subseteq I$ with $|I_t| = |I_t^{'}| = t$, and $J = \{j\}$ we have

$$Pr[I_t, \{j\}] = Pr[I_t^{'}, \{j\}] \ .$$

For this exercise you can assume that the values v_{ij} are pairwise distinct.

Exercise 12.2. (4 Points)

We consider a Forest Auction with n bidders in a graph G. We assume all bidders have pairwise distinct values. We denote the size of each spanning tree in G by k and use the Random Threshold Mechanism. Let S denote the output of the Random Threshold Mechanism and $m_i(S)$ the number of bidders in S with values at least $v_i/2$, where v_i is the i-th highest value of the bidders in the optimal spanning tree. Prove that

$$\mathbb{E}[m_1(S)] \geq \frac{1}{8(\lceil \log k \rceil + 1)} \ .$$

Exercise 12.3. (3+3 Points)

In the Random Threshold Mechanism we choose some $j \in \{0, \dots, \lceil \log k \rceil\}$ uniformly at random.

a) Show that there is a family of instances of Forest Auctions (with $k > 1$), where the non-randomized variant of the Threshold Mechanism, where we always choose $j = 0$, is not $\mathcal{O}(\log k)$-competitive.

b) Show that there is a family of instances of Forest Auctions (with $k > 1$), where the non-randomized variant of the Threshold Mechanism, where we always choose some fixed $j \in \{1, \dots, \lceil \log k \rceil\}$, is not $\mathcal{O}(\log k)$-competitive.

The exercise sheets and more information about the course can be found at `http://algo.cs.uni-frankfurt.de/lehre/agt/winter1920/agt1920.shtml`

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