Algorithmic Game Theory

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Exercise Sheet 8

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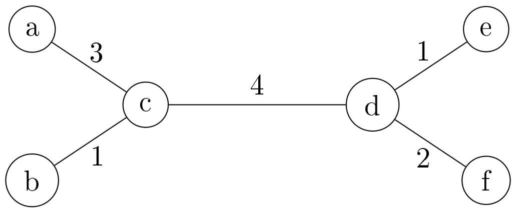
Please hand in your solutions until Tuesday, January 14, 10:15h, in H9 or in the letterbox between rooms 114 and 115, R.M.S. 11-15.

Exercise 8.1. (3 Points)

Santa Claus wants to auction one present with $n > 2$ bidders. We assume that no two bids are the same. Santa assigns the present to the bidder with the highest bid. The winner of the present pays the k- th highest bid for some $k \in \{2, 3, 4, \ldots, n\}$. The other bidders do not pay anything. For which k is this mechanism incentive compatible? Why?

Exercise 8.2. (4+3 Points)

In a *path auction* there is an underlying network given as an undirected graph $G = (V, E)$ with edge capacities $c_e \in \mathbb{N}$ for all $e \in E$. Every bidder $i \in N$ has a desired path $P_i \subseteq E$, a demand $d_i \in \mathbb{N}$ and a valuation $v_i \in \mathbb{R}_+$ for getting allocated. Based on the bids of the players, the auctioneer decides which player is allowed to use his desired path and for which price. However, the allocation of paths to bidders needs to be feasible in the following sense: The sum of demands of players using an edge e may not exceed the capacity c_e. We do not allow fractional assignments.



Bidder	Path	Demand d_i	Value v_i
1	(ac, cd)	2	12
2	(bc, cd, de)	1	6
3	(cd, df)	3	10
4	(ed, df)	1	3

a) Consider the following allocation rule:

- Order the bidders by their bid per unit of demand $\frac{b_i}{d_i}$. Break ties arbitrarily.
- Iterate over the ordered bidders and greedily allocate their path if possible.

Can this allocation rule be used in an incentive-compatible mechanism for path auctions? Prove your answer. If it is 'no' give an IC mechanism for path auctions.

b) Apply the mechanism of a) to the given example network. Compute the outcome and the prices for all bidders. Does the allocation rule always maximize social welfare?

Exercise 8.3. (4 Points)

We concluded the proof of Myerson's Lemma by giving a *proof by picture* that coupling a monotone and piece-wise constant allocation rule x with the payment formula

$$p_i(v_i, v_{-i}) = v_i x_i(f(v)) - \int_{t_i^0}^{v_i} x_i(f(t, v_{-i}))dt \ ,$$

yields an incentive compatible mechanism. Where does the proof by picture break down if the piece-wise constant allocation rule x is not monotone? In order to do so, give an example for a non-monotone allocation function and the payment formula above such that truthful bidding is not a dominant strategy for at least one player. Explain where the proof idea breaks down.

Exercise 8.4. (4 Points)

Consider the following Combinatorial Auction. You are an owner of a store and you want to auction the space of three racks in your store during the pre-Christmas sales. The bidders are companies that may place some goods on the racks in order to sell them. Rack A is very big, where racks B and C are small, but rack B is in a better position then racks A and C. You have 4 companies C1, C2, C3, C4 taking part in your auction with the following true valuations.

Companies \Racks	$\emptyset$	A	B	C	AB	AC	BC	ABC
C1	0	1	1	1	1	1	1	1
C2	0	2	4	2	4	2	3	3
C3	0	2	0	0	5	4	0	7
C4	0	0	0	0	3	3	3	5

We apply a social welfare maximizing IC mechanism, i.e. you choose the payment formula given by the VCG auction. Assuming true bidding, say which company is allocated which rack and calculate the prices.