Algorithmic Game Theory

Winter Term 2019 / 2020

Prof. Dr. Martin Hoefer, Dr. Daniel Schmand

JOHANN WOLFGANG GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Algorithmen und Komplexität
Institut für Informatik

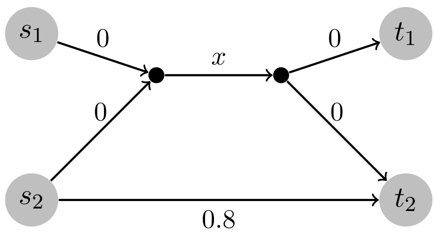
Exercise Sheet 6

Publication: Nov 26, 2019
Solutions Due: Dec 03, 2019

Please hand in your solutions until Tuesday, December 03, 10:15h, in the ***letterbox*** *between rooms 114 and 115, R.M.S. 11-15. There will be* ***no lecture*** *on December 03!*

Exercise 6.1. (1 + 4 + 2 Points)

Consider the following multi-commodity Wardrop game. The graph and the latency functions are given as depicted below. There are two commodities, (s_1, t_1) with $r_1 = 0.4$ and (s_2, t_2) with $r_2 = 0.6$.



a) Consider the following flow: All players of commodity 1 use the only possible path. The players of commodity 2 split as follows. A total mass of 0.4 use the direct edge from s_2 to t_2 and the rest use the upper path.

   Calculate the social cost for the given profile.

b) Calculate a Wardrop equilibrium, the socially optimal flow, and the prices of anarchy and stability.

c) Now assume that the players in every commodity form a coalition and chose their paths in such a way that they minimize the total cost of the population in their respective commodity (this model is also called *atomic-splittable routing game*). What is the equilibrium flow in this situation and what is the social cost of the equilibrium flow?

Exercise 6.2. (5 Points)

We are given a Wardrop game with latency functions $d_e(x) = a_e x^2 + b_e x + c_e$ for some non-negative parameters a_e, b_e, c_e, for all $e \in E$. Show that the price of anarchy is at most $\frac{3\sqrt{3}}{3\sqrt{3}-2}(\approx 1.62)$.

Hint: Argue in 1-2 sentences that -by a transformation of the network- you can assume the cost functions to be monomials. Then, argue separately for

i) $d_e(x) = a_e x^2$,

ii) $d_e(x) = b_e x$,

iii) $d_e(x) = c_e$,

that the smoothness condition holds for parameters $(1, \frac{1}{\sqrt{3}} \cdot \frac{2}{3})$. *Finally, derive the PoA.*

Exercise 6.3. (3 + 3 Points)

a) Construct an equal-sharing game with a pure Nash equilibrium S and a socially optimal state S^* such that

$$\frac{\text{cost}(S)}{\text{cost}(S^*)} = n \, .$$

Argue in one sentence why S is a pure Nash equilibrium.

b) Prove that equal-sharing games are $(n, 0)$-smooth, i.e. that the price of anarchy for coarse-correlated equilibria is at most n.

The exercise sheets and more information about the course can be found at `http://algo.cs.uni-frankfurt.de/lehre/agt/winter1920/agt1920.shtml`

Email: `mhoefer@cs.uni-frankfurt.de, schmand@em.uni-frankfurt.de`