Algorithmic Game Theory

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Exercise Sheet 2

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Please hand in your solutions until Tuesday, November 05, 10:15h, in H9 or the letterbox between rooms 114 and 115, R.M.S. 11-15.

Exercise 2.1. (5 Points)

We are given a 2 player-zero sum game with the following utility matrix for player 1.

$$A = \begin{bmatrix} 4 & -2 & -1 & 2 & 3 \\ -2 & 7 & -1 & 1 & -1 \\ -3 & 4 & -2 & 0 & 2 \\ 2 & 1 & 1 & -5 & -2 \end{bmatrix}$$

Calculate a mixed Nash equilibrium by using the LPs from the lecture. Please hand in the solution to this exercise including your code you used to solve the LPs.

You can solve the LPs with your favorite LP-solver, for example with the SCIP Optimization Suite. You can find an example on how to model an LP in ZIMPL format on the course webpage. You can edit the example with any text-editor. For optimizing you can use SCIP or any other LP solver. Installers for Windows, Mac and Linux can be found at `http://scip.zib.de/#download`. After the installation, open SCIP in the console, read your problem by typing `read example.zpl`, optimize by typing `optimize` and look at the solution by `display solution`. It will show you the values of all non-zero variables. If you have any questions, feel free to write us an Email or to come to our office hours, as always.

Exercise 2.2. (2+3 Punkte)

Suppose (x^1, y^1) and (x^2, y^2) are two mixed Nash equilibria in a 2-player zero-sum game.

a) Are (x^1, y^2) and (x^2, y^1) mixed Nash equilibria? Prove your answer.

b) Show that the state (x', y') with

$$\begin{aligned} x'_j &= (x^1_j + x^2_j)/2 \quad \text{for all } j \in \{1, \ldots, m_1\} \\ y'_j &= (y^1_j + y^2_j)/2 \quad \text{for all } j \in \{1, \ldots, m_2\} \ . \end{aligned}$$

is a mixed Nash equilibrium.

Exercise 2.3. (1+3 Points)

For an exact potential game Γ, show that

a) there are infinitely many exact potential functions for Γ, and that

b) two different exact potential functions Φ_1 and Φ_2 for Γ there is a constant c_{12}, such that

$$\Phi_1(S) - \Phi_2(S) = c_{12} \quad \text{for all states } S \in \Sigma.$$

Exercise 2.4. (3 Points)

The following game is known as the *Pollution Game*. There are n players in this game and each player represents a country. For simplicity, we assume that each country has the following two choices: Either it agrees to set industry standards such that its pollution is controlled, or not. Mixed strategies are not allowed. Agreeing to control pollution costs 3 for the country to set the industry standards. Each country that does not agree to control pollution adds 1 to the cost of all countries, including itself. Formally, every cost function $c_i : \Sigma \to \mathbb{Z}$ is given by

$$c_i(S) = \begin{cases} 3 + |\{j \in N \mid j \text{ does not agree}\}| & \text{if } i \text{ agrees,} \\ |\{j \in N \mid j \text{ does not agree}\}| & \text{if } i \text{ does not agree.} \end{cases}$$

Construct an exact potential function Φ for this game. Prove that it is an exact potential function. Be prepared to argue orally in which way this shows the existence of a pure Nash equilibrium.