Algorithmic Game Theory
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Exercise Sheet 1

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Please hand in your solutions until Tuesday, October 29, 10:15h, in H9 or the letterbox between rooms 114 and 115, R.M.S. 11-15.

Exercise 1.1. (2+2 Points)

	E	F	G	H
A	8 8	9 6	8 6	9 8
B	9 9	8 7	5 8	9 7
C	6 7	6 2	9 6	8 4
D	3 9	8 8	5 6	6 8

Consider the 2-player game given by the matrix above. Calculate all

a) dominant strategies of the players,

b) pure Nash equilibria.

Exercise 1.2. (3+3+3 Points)

A strategy $s_i \in S_i$ of player i is called **strictly dominated** by strategy $s'_i \in S_i$, if s'_i is always strictly better then s_i, i.e. for all s_{-i} we have

$$c_i(s'_i, s_{-i}) < c_i(s_i, s_{-i}).$$

	W	X	Y	Z
A	3 / 2	4 / 6	1 / 3	6 / 9
B	2 / 3	3 / 2	5 / 6	1 / 8
C	3 / 1	1 / 2	5 / 5	2 / 4
D	4 / 7	5 / 4	3 / 2	2 / 5

a) Iteratively, eliminate all strictly dominated strategies in the cost matrix given above. Do this until there are no strictly dominated strategies left. Write down your cost matrix after each step.

b) Calculate a mixed Nash equilibrium in the reduced cost matrix.

c) Prove the following statement: In all matrix games, there is a one-to-one correspondence between mixed Nash equilibria in the original and the reduced game, when applying the reduction procedure described above.

Exercise 1.3. (2 + 2 + 2 Points)

We generalize the Sperner-lemma to squares in the following way: We consider a square S that is subdivided into smaller squares by a grid of lines parallel to the edges of the original square. The vertices of the subdivision are the points of intersection of the lines. A Sperner-coloring of S is a coloring of vertices that fulfills the following properties:

- The four corners of the outer square are colored with the colors red, blue, orange, purple in clockwise order.
- Every point on a side of S is colored with one of the two colors of the endpoints of the corresponding outer line.
- Vertices in the interior of S are colored arbitrarily in one of the four colors.

We call edges between red and blue vertices the *doors*. Doors at the boundary of S are called *entrances*. Show the following properties for a Sperner-coloring of a square S:

a) There is an odd number of entrances.

b) There is at least one small square with at least 3 different colors.

c) There is an odd number of small squares, with at least 3 different colors and exactly one door.

The exercise sheets and more information about the course can be found at `http://algo.cs.uni-frankfurt.de/lehre/agt/winter1920/agt1920.shtml`

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